

Answers to Exercise 3

A. Solve the following indefinite integrals.

1. $\int 5dx \Rightarrow F(x) = 5x + c$
2. $\int 3x^2 dx \Rightarrow F(x) = x^3 + c$
3. $\int (4x^3 - 3x^2 + 1)dx \Rightarrow F(x) = x^4 - x^3 + x + c$
4. $\int \frac{1}{x} dx \Rightarrow F(x) = \ln x + c$
5. $\int e^{x+1} dx \Rightarrow F(x) = e^{x+1} + c$
6. $\int (3\sqrt[3]{x^2} + 1)dx \Rightarrow F(x) = \frac{9}{5}x^{5/3} + x + c$
7. $\int 4x^3 e^{x^4} dx \Rightarrow F(x) = e^{x^4} + c$
8. \int
9. $\int \frac{x}{x^2 + 4} dx \Rightarrow F(x) = \frac{1}{2} \ln(x^2 + 4) + c$
10. $\int \frac{x^2 - 1}{x^3 - 3x} dx \Rightarrow F(x) = \frac{1}{3} \ln(x^3 - 3x) + c$

B. Evaluate the following indefinite integrals, where the initial condition is x=0, y=0.

1. $\int 4dx \Rightarrow F(x) = 4x$
2. $\int (1 - x)dx \Rightarrow F(x) = x - \frac{x^2}{2}$
3. $\int (3x + 3)dx \Rightarrow F(x) = \frac{3x^2}{2} + 3x$
4. $\int (x^{\frac{1}{2}} + 3x^{-\frac{1}{2}})dx \Rightarrow F(x) = \frac{2}{3}x^{3/2} + 6x^{1/2}$

C. Find the area under the curve for the following definite integrals.

1. $\int_0^2 4dx \quad A = 8$
2. $\int_{-1}^1 (1 - x)dx \quad A = 2$
3. $\int_0^2 (x - 3)dx \quad A = 4$
4. $\int_{-1}^1 (x^3 + x + 6)dx \quad A = 12$
5. $\int_1^2 \frac{1}{x} dx \quad A = 0.693$
6. $\int_1^2 e^x dx \quad A = 4.671$

D. This is similar to B

E. If the MC is given as the following function and the Fixed Cost is 43,

$$MC = 32 + 18Q - 12Q^2$$

find;

- i) the total cost function,
- ii) the average cost function,
- iii) the variable cost function,

$$MC = 32 + 18Q - 12Q^2$$

$$TC = \int MC dQ = \int (32 + 18Q - 12Q^2) dQ \Rightarrow TC = 32Q + 9Q^2 - 4Q^3 + 43$$

$$AC = \frac{TC}{Q} = 32 + 9Q - 4Q^2 + \frac{43}{Q}$$

$$VC = 32Q + 9Q^2 - 4Q^3$$

F. Given the following supply and demand functions, find

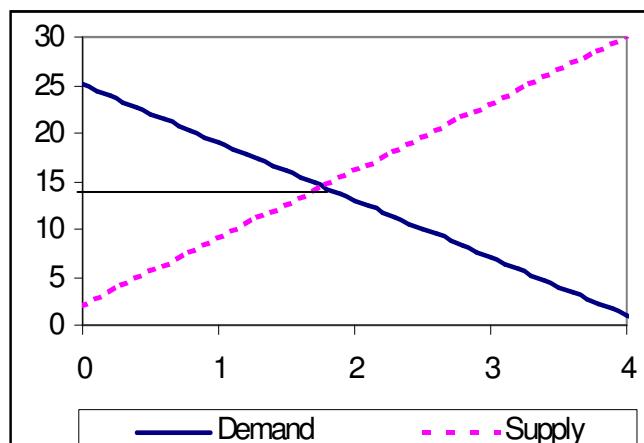
$$P_D = 25 - 6Q \quad P_S = 2 + 7Q$$

- i) the equilibrium price and demand,
- ii) show graphically, consumer's and producer's surplus,
- iii) calculate consumer's and producer's surplus,

$$\begin{cases} P = 25 - 6Q \\ P = 2 + 7Q \end{cases} \Rightarrow Q_E = \frac{23}{13}, \quad P_E = \frac{187}{13}$$

$$PS = \frac{23}{13} * \frac{187}{13} - \int_0^{23/13} (2 + 7Q) dQ \Rightarrow PS = 25.45 - \left[2Q + \frac{7}{2}Q^2 \right]_0^{23/13} = 10.95$$

$$CS = \int_0^{23/13} (25 - 6Q) dQ - \frac{23}{13} * \frac{187}{13} \Rightarrow CS = \left[25Q - 3Q^2 \right]_0^{23/13} - \frac{23}{13} * \frac{187}{13} = 9.39$$



G. Given the following demand and supply functions for cruise passenger services in a region,

$$\text{Demand Equation } P_d = 10 e^{-\frac{Q}{10}}$$

$$\text{Supply Equation } P_s = e^{\frac{Q}{10}}$$

- i) Find the equilibrium freight rate,

$$P_d = P_s \Rightarrow 10e^{-\frac{Q}{10}} = e^{\frac{Q}{10}}$$

$$\ln 10 - \frac{Q}{10} = \frac{Q}{10} \Rightarrow Q_e = 5 \ln 10 = 11.513 \Rightarrow P_e = 3.162$$

- ii) Find the consumers' surplus and producers' surplus,

$$PS = (11.513)(3.162) - \int_0^{11.513} e^{\frac{Q}{10}} dQ \Rightarrow PS = 36.404 - \left[10e^{\frac{Q}{10}} \right]_0^{11.513} = 14.781$$

$$CS = \int_0^{11.513} 10e^{-\frac{Q}{10}} dQ - (11.513)(3.162) \Rightarrow CS = \left[-100e^{-\frac{Q}{10}} \right]_0^{11.513} - 36.404 = 68.377$$

- iii) Show the consumers' surplus and producers' surplus on a graph.

